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VIEWPOINT

Mesoscopic transport revisited

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Abstract

Having driven a large part of the decade's progress in physics, nanoelectronics is now passing from today's realm of the extraordinary to tomorrow's commonplace. This carries the problem of turning proofs of concept into practical artefacts. Better and more sharply focused predictive modelling will be the ultimate guide to optimizing mesoscopic technology as it matures. Securing this level of understanding needs a reassessment of the assumptions at the base of the present state of the field. We offer a brief overview of the underlying assumptions of mesoscopic transport.

1. Context of mesoscopic transport

At the mesoscopic scale and even more so below that, the main challenge in electron-transport theory is how to characterize the intrinsic properties of a structure when surface-to-volume ratios are no longer negligibly small. The notion of an 'intrinsic' property becomes ill-defined when the interfaces to the external circuitry and the thermal surroundings compare in size with the device that is coupled to them.

The issue is highlighted by a simple example. What, in practice, is the real physical size of a sub-micrometre conducting channel when the bulk scattering length in the constituent material is measured in microns? There is no reason to take the electrically relevant 'length' to be, say, the lithographic one. How can one tell where the device proper ends and its interface regions begin?

Such questions must first be addressed operationally. Only after an answer suggests itself by what is actually seen in measurement does it make sense to seek a general picture of transport. As well as the dynamics, a well-posed transport theory has to subsume the topological characterization of the device (with interfaces) through the *boundary conditions* that effectively define what is an 'open system' within quantum kinetics.

The actual physical definition of a mesoscopic device poses a non-trivial challenge within this field, conditioning its future development as tightly as its present one. Moreover, even deeper issues emerge whose physical expression and

action are not negotiable. By that we mean that it remains crucial always to give a full accounting of the global electrodynamic properties of a conducting system, in its entirety. The way of including these global properties is always the same whether the device itself is classical or quantum, macroscopic or mesoscopic.

To analyse a driven conductor as a structure of finite extent is, by that act of abstraction, to specify it as an open system. The constraints of microscopic *and* global charge conservation both apply. The former alone, through the continuity equation, cannot guarantee the latter when there are open boundaries to act as effective generators and absorbers of electron flux.

Here it is the interlinked topology of driving fields and currents that dictates the electromagnetic response. Thus, any purely local specification of electrical quantities (such as the chemical potential) is not enough to determine a globally invariant quantity such as the current. For example, let us posit that any current is to be generated by a difference of chemical potential, set up between the leads of a conductor. Then such a mechanism should always explicitly guarantee the asymptotic stability of the leads *far from the active device*. In other words, perfect screening (strict long-range neutrality) must be respected.

In this paper we adopt a definite historical position: that all of the conceptual and formal tools necessary for meso and nanoscopic transport already exist within the developments of the past. We have in mind the quantum-kinetic framework typified by Fermi liquid theory, the Kubo formula, and the Kadanoff–Baym–Keldysh non-equilibrium approach: not only readily accessible but immediately applicable as the

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definitive benchmarks for any and all mesoscopic calculations at the present.

In a normal, open system, finite resistance is microscopically understandable if and only if there is a mechanism for dissipation of the electrical energy imparted to the system. This physical necessity is not addressed by models that admit *elastic* transmission as the sole, exclusive, scattering process.

It is worthwhile to recall the sharp qualitative distinction between tunnelling and metallic conduction. In tunnelling à la Bardeen, the conductivity as a function of barrier width d is

$$\sigma \sim \exp(-2d\sqrt{2V_0 - k^2}),$$

V_0 being the barrier height in units of \hbar^2/m and k the wavenumber of the incoming carrier. The exponentially decaying nature of barrier tunnelling is clear.

As commonly applied, the Landauer conductance formula has had the widest appeal of any transport model in mesoscopics and has been taken as the ultimate generalization of the earlier Bardeen picture of tunnelling transmission. But the basic exponential form of the transmission probability is now freely replaced with an arbitrary quantity (so long as it does not exceed unity). See the comments by Frensley [3] as well as those in [1].

On the other hand, metallic conduction presents a picture of transport that is physically very different to tunnelling. It is inherently tied to the many-body response of carriers in free, extended states and involves *collective* displacements of the Fermi sea. For this reason, more recent attempts to import inelasticity and dissipation post hoc into *one-body* coherent-transmission models of resistance have yet to demonstrate their conformity with conservation laws.

Dissipation depends on the strength of electron–hole pair relaxation; that is, it is the decay of the current–current correlation function. By contrast, some authors attempt to mimic this via decay of the single-particle strength alone, which compromises particle conservation. In short: for any regime where dissipation has a role, a conserving many-body description is crucial. With that central concept in mind, it is clear that there already exists a completely established microscopic context for assessing the more recent theoretical innovations.

We place specific focus in this paper on the notion of mesoscopic electrical conductance. First, mesoscopic conductance is the key to many other electronic phenomena at this scale and below. Second, its theoretical basis is the one most studied with the current set of methodologies.

In section 2 we revisit the conductance theory widely associated with Landauer and Büttiker. In section 3 we examine its leading experimental confirmations with an eye to the conceptual issues reviewed above. Of special relevance is the interplay of theory and the two complementary experimental methods known as ‘two-terminal’ and ‘four-terminal’ measurements. Finally in section 4 we summarize the essential lessons that can be drawn for the present situation in mesoscopic analysis.

2. Landauer conductance: theory and experiment

The physical resistance of a conductor takes its meaning only through measurement of current and voltage directly through the contacts to the device under test. Naturally the best contacts for such work are those that are seen to have the smallest disturbing effects on the inherent resistance of the sample.

For a uniform one-dimensional (1D) wire the Landauer model interprets the conductance in terms of the transparency of the wire in series with its contacts (see for example [1, 2]). The whole is conceived as a quantum barrier. For a perfectly transmitting barrier the transmission function, namely the ratio of outgoing to incoming carrier flux, attains its maximum of unity. In other cases there is reflection at the source interface, and the transmission ratio falls below its maximum.

The Landauer formula for the ‘two-terminal’ conductance (see below) is

$$G_2 = G_0 \mathcal{T} \leq G_0; \quad G_0 = \frac{2e^2}{h} \approx 77.5 \mu\text{S}. \quad (1)$$

The transmission function is \mathcal{T} and G_0 is Landauer’s quantum: the conductance of a perfectly uniform, perfectly one-dimensional metallic conductor constituting a perfectly transmissive quantum barrier. Any actual scattering by the 1D barrier is represented by \mathcal{T} . The transmission theory of conductance is, at base, a phenomenology. This is because the governing parameter \mathcal{T} is never specified microscopically, as an actual physical object within the theory. The theory leaves to others the central task of computing the physical answer essential to the character of the transmission function, and to any practical computation. See, for example, the analysis by Agraït *et al* [4].

As a practical fact \mathcal{T} can indeed include any scattering, whether elastic or inelastic, if it is calculated starting from the conserving, many-body microscopics of a conductive system. In particular the action of dissipation in mesoscopic transport can be directly shown via the Kubo formula, from which the characteristic Landauer conductance steps emerge [5].

Perhaps it is appropriate to make an historical remark here. In Landauer’s very first publication on (classical) resistivity due to localized scattering in metallic conduction [6], he included in \mathcal{T} any process that could disturb the otherwise ballistic motion of single carriers. Landauer explicitly recognized the action of inelastic contributions to resistance; it was other writers who went on, well after him, to claim an exclusive role for coherence in mesoscopics. As a result, Landauer himself felt obliged to comment on the conceptual gap that opened up between his original conception and its widespread reinterpretations. See his opening comment in [7] which annotates the reprint of his seminal paper, [6]. Landauer notes further how few people, in the present day, still seem to take any time to read the 1957 work.

2.1. Two-terminal measurements

The Landauer formula refers to the expected outcome for conductance when the two probes of the voltmeter span the entire structure, consisting of the device and its two interfaces

in series. In effect, the voltage probes coincide with the pair of terminals through which the current enters the sample and returns to the macroscopic supply circuit. It is referred to as a *two-terminal* arrangement.

In this model it is always assumed that the system presents a strictly elastic barrier to electron flow; indeed, the barrier may be ideally transparent. In any case, dissipation cannot occur within this description, regardless of how the conductor may be probed.

Nevertheless, all normal conductors always entail dissipation. It follows that there must be a corresponding, steady electrical energy loss associated with the Landauer conductance—as with every resistive system—occurring at the familiar well-defined rate

$$P = IV = G_0 \mathcal{T} V^2. \quad (2)$$

Where is P actually dissipated? The power must be discharged somewhere. Yet it cannot be released within the core of the 1D structure; nor can it be released at the interfaces, because the Landauer theory of two-terminal transmission excludes dissipation in the whole barrier and in particular at ideal transmission, $\mathcal{T} = 1$. In fact, perfect conduction has been achieved and widely reported [8–10].

Experiments have thus fully confirmed the existence of a Landauer conductance. However, they provide no insights into the concrete issue of physical dissipation. So an apparent ‘missing link’ remains between equations (1) and (2).

2.2. Four-terminal measurements

In principle, one may be able to eliminate the interface series resistances entering into the two-terminal conductance, so accessing the actual resistance of a ballistic device. If a pair of perfect voltage probes were to be placed across the 1D conductor at the heart of the structure, they would directly read off the local electrostatic potentials and provide the quantities needed to deduce the intrinsic resistance. The additional inner probes should be spatially separated from the current-supply contacts, which remain asymptotically far away: a conceptual arrangement known as a *four-terminal* measurement set-up [1]⁴.

The essential requirement on all internal voltage probes is ‘non-invasiveness’. That is: the probes must not substantially disrupt the pattern of carrier flux within the device before it is probed in this way. Otherwise it would be impossible to deconvolve the probes’ influence uniquely from the raw measurements. What makes the mesoscopic scale notoriously challenging is precisely that interventions with probes tend strongly to modify the internal landscape, putting any data at risk of being too sample-specific to give systematic clues to the transport physics.

Assuming the feasibility of non-invasive probes, the second hurdle is that one does not know *a priori* where to place them because the physical distinction between the ‘true’ mesoscopic conductor and its interfaces is ill-defined. The

margin of uncertainty in locating the probes is the scale of the screening regions associated with the Landauer dipole⁵. Nevertheless, if the voltage probes genuinely do not disrupt current flow, one can envisage making successive tests to systematically relocate them until measurements no longer report any voltage drops from locally changing densities in the interface regions. One could then argue operationally that the ‘inner’ conductor had been marked out.

Granted all of those conditions, there is a Landauer four-terminal conductance formula for expressing the true, intrinsic conductance of a uniform one-dimensional wire. Thus

$$G_4 = G_0 \frac{\mathcal{T}}{1 - \mathcal{T}}. \quad (3)$$

The physical explanation of equation (3) is detailed elsewhere [1]. The formula implicitly adjusts the Fermi levels *immediately adjacent* to the left and right ‘boundaries’ of the inner conductor, by removing the potential contribution from the Landauer dipole.

Büttiker [13] originally derived a general four-terminal conductance formula in terms of the partial transmission probabilities \mathcal{T}_{ij} between all pairs i, j of available terminals (current leads have $i = 1, 2$, voltage leads $j = 3, 4$):

$$G_4 \equiv G_2 \frac{(\mathcal{T}_{31} + \mathcal{T}_{32})(\mathcal{T}_{41} + \mathcal{T}_{42})}{\mathcal{T}_{31}\mathcal{T}_{42} - \mathcal{T}_{41}\mathcal{T}_{32}}. \quad (4)$$

As long as non-invasiveness applies, equations (3) and (4) are equivalent. We will return to this formula shortly.

3. Implications

The additional information in a four-terminal measurement can be combined with a two-terminal measurement to give the series contact resistance, due strictly to the interfaces. In a perfect quantum barrier where $\mathcal{T} = 1$, equation (3) clearly implies infinite conductance. The intrinsic wire has no resistance. Thus the contribution to the ideal two-terminal conductance $G_2 = G_0$, can come only from the interfaces.

We now discuss a few examples out of the large experimental literature, and how we can come to understand these results with the help of available theories. It is not inappropriate to say that certain observed features remain unexplained, and do need further laboratory investigations.

3.1. Perfect conduction

An essentially perfect four-terminal measurement was reported in the remarkable experiment by de Picciotto *et al* [14]. Using a very high-quality 1D sample produced by cleaved-edge overgrowth and built-in voltage probes of low invasiveness,

⁵ Since the prevailing view of conductance is strictly for single, coherent, non-interacting carriers, Landauer’s dipole is attributed, not to the inhomogeneous Coulomb screening response at the boundaries (see [12]), but only to a band-structure induced bottleneck effect on coherent transmission. The electron wavefunction is envisaged to go from its extended bulk form in the source and drain contacts, to highly constricted form in the 1D conductor whose density of states is greatly reduced. The current, of course, is unaffected and is everywhere constant through the system; but the Landauer dipole reduces it by reducing the overall transmission ratio \mathcal{T} .

⁴ An early substantial conflict regarding application of the expression for G_4 over against G_2 was first resolved by [11].

they detected essentially zero resistance in their core conductor, so that the transmission factor was essentially unity. Their direct four-terminal findings complement the earlier purely two-terminal data by van Wees *et al* [8], demonstrating perfect Landauer quantization of G_2 after removal of ‘parasitic’ access resistances.

De Picciotto *et al* also displayed the two-terminal conductance of their sample. A noticeable shortfall about 7% below G_0 is observed in the height of the sub-band steps in G_2 . According to equation (1) this implies a non-ideal transmission factor: $\mathcal{T} = 0.93$. On the other hand, equation (3) and their zero-resistance data imply $\mathcal{T} = 1$.

Other four-probe experiments have been performed on mesoscopic samples:

- Kvon *et al* [15] reported a small but finite intrinsic four-terminal resistance G_4^{-1} in a diffusive mesoscopic wire, where the bulk elastic-scattering mean free path is comparable to, possibly less than, the sample length so that transmission is no longer ballistic. The presence of a stepwise structure is nevertheless evident in the corresponding plots of G_2^{-1} , although coherent transport does not apply in their regime. It is still possible, however, to subsume these results within the Landauer picture.
- A four-terminal experiment somewhat closer to [14] was performed by Reilly *et al* [16] in a conventionally fabricated, low-disorder wire built on two-dimensional GaAs heterojunction material. Unlike the zero-resistance results [14], the four-terminal conductance of the gate-defined structures reported here (down to a 1D wire of zero nominal length) is neither infinite nor even excessively large. Instead, one sees in the raw four-terminal conductance a series of perfect Landauer steps that seem to follow equation (1) rather than equation (3), despite the radical structural difference between G_2 and G_4 within the transmission theory. Curiously, it is reported [16] that this holds true even in zero-length channels.

3.2. Negative intrinsic resistance

A very different light on quantum ballistic transport has been shed by recent multi-terminal experiments [17, 18]. In each case certain specific resistances, measured internally to the device-plus-lead assembly, become negative. What is addressed here is an ‘absolute’ resistance V/I at the level of Ohm’s linear law; decidedly not some differential resistance dV/dI (well known to become negative in strongly nonlinear transport, as in resonant-tunnelling structures) which—in sharp contrast to absolute resistance—is *not* constrained by the thermodynamics of power dissipation, namely the relation $P = I^2 R > 0$. Further, if $R_4 = 1/G_4$ is negative then equation (3) also shows that $\mathcal{T} > 1$.

The intrinsic resistance measured by Gao *et al* [17] is theoretically identical with R_4 deduced from Büttiker’s generic description of multi-probe conduction, equation (4), and therefore also with Landauer’s equivalent version equation (3) at low invasiveness [1]. According to [17] it is the difference of two products of partial probabilities, $\mathcal{T}_{31}\mathcal{T}_{42} - \mathcal{T}_{41}\mathcal{T}_{32}$, which conceivably explains their negative intrinsic resistance [13]. If so, however, this entails a set of puzzling contradictions [19]:

- if $R_4 < 0$ then the power dissipation in the device proper is $P = I^2 R_4 < 0$ and the device must spontaneously be giving up energy to the rest of the circuit,
- if $R_4 < 0$ and $\mathcal{T} > 1$ then unitarity (probability) is not conserved, even though the Landauer formula (3), equivalent to the Büttiker formula (4), is predicated on unitary single-electron propagation.

The second experiment by Kaya and Eberl [18], examined a closely related quantity, the three-terminal resistance, with the conclusion that this too can be negative. In this set-up only one voltage probe is placed inside the structure, while its companion probe remains located on the far current lead.

The implications here are the same as above. The multi-probe Büttiker model of transmission strictly precludes the three-terminal resistance from becoming negative. If the latter theory is correct under the appropriate mesoscopic conditions, the data seems once more to describe effects that are not consistent with the theory.

We may sum up the negative-resistance outcomes as follows. Under physically reasonable conditions, such data imply the violation of particle and energy conservation in normal ballistic systems. Within such systems the laws of linear transport ought to hold, internally and externally and regardless of scale. Unless the theory of coherent multi-probe transport is wrong, the measurements must be open to other interpretations than the notion of ohmic resistances’ being negative (but otherwise plain and ordinary).

3.3. The 0.7 anomaly

Finally we comment on the ‘0.7 anomaly’. In this phenomenon, a step in the two-probe conductance (anywhere in height between 0.5 and 0.9 G_0 but quite often $\approx 0.7G_0$), briefly precedes the onset of the normal, full-sized Landauer step as the carrier density is increased above the sub-band threshold.

Such behaviour is not predicted by the non-interacting transmission model of two-probe conductance. Therefore it is presumed to be an effect of many-body correlations. The existing 0.7 literature reveals a continuing unresolved debate, not only between proponents of the two main theoretical lines but also among the outcomes of various reported experiments. Thus, for instance, if one carefully reads the contributions to the recent special issue of *J. Phys.: Condens. Matter* [20] one can form the reasonable impression that the 0.7 anomaly is shrouded in mystery—and remains so.

In one picture of the anomaly, it is due to the spontaneous opening of a spin-polarized energy gap, with the sub-band of spin states at lower energy being responsible for the 0.7 precursor step. This approach nevertheless relies heavily on the accepted phenomenology of conductance as dissipationless transmission, so that the issues attaching to the central place of energy loss must also apply here.

In the competing quite different picture, a local spin-polarized impurity in the 1D conductor generates a Kondo-like resonance at an energy near the bottom of the 1D conduction band. This somewhat enhances the density of states, and hence the carrier flux, just before the Fermi level accesses

the normal conduction-band threshold (where the standard Landauer conductance is observed). There is no reason to believe that this scenario, at least, would have a problem being cast within a microscopic framework such as that of the Kubo response formula.

Ultimately, of course, only firmly validated measurements will distinguish among theoretical alternatives. In the case of the spin-polarization picture, there should be a channel-length dependence of the observed size of the anomaly [21]. This is not so in the Kondo picture, which posits a much more localized exchange-interaction effect. But various sets of experimental data on the length dependence, obtained to date, can be cited in support of either possibility. In other words: so far, there are no firm results to show.

4. Conclusion

The heart and soul of a mesoscopic theory reside in its physical credibility. Securing that credibility requires sustained, diligent and harmonious work by experimentalists and theorists. As we have shown above, however, careful study of some contemporary experimental works in mesoscopics inspires less than full confidence in their potential to clear the way to better mesoscopic modelling.

There are enough internal and mutual conflicts in the interpretation of some recent observations to cloud the basic physical issues rather than clarify them. The theoretical task is not made easier when experimental works, appearing in the record, contradict one another and even themselves—not to mention the entire spectrum of available mesoscopic models, widely accepted or otherwise.

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